Prediction of the Effect of Weeping on Distillation Tray Efficiency

The reduction in distillation tray efficiency caused by uniform weeping of liquid from the tray has been calculated. Numerical results are presented for all three Lewis cases over the range of variables of interest to tray designers. Analytical equations are given for Lewis' cases 2 and 3 for plug flow of liquid on the tray.

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SCOPE

One consequence of the present economic climate is that many distillation columns are operating at less than their design capacity. At reduced vapor loads, weeping becomes a problem in trayed columns because it reduces tray efficiency and so adversely affects the separation achieved by the column. To maintain the required separation, it is possible to overreflux. This reduces the number of theoretical trays required and also, by maintaining a high internal vapor load, reduces the tendency to weep. However, overrefluxing increases energy consumption.

To determine the best strategy of operation at reduced throughputs, it is important to predict the reduction in tray efficiency which results from weeping. Previous work, by Kageyama, deals only with the case where the vapor is completely mixed between trays (Lewis Case 1). This is a reasonable assumption only for small pilot-plant-size columns. The objective of the present study was to extend the analysis so as to be applicable to industrial columns by making the assumption that the vapor is unmixed between trays. In addition, both alternate direction (Lewis Case 3) and parallel liquid flow on successive trays (Lewis Case 2) have been considered.

CONCLUSIONS AND SIGNIFICANCE

In the presence of weeping, distillation tray efficiency depends on the fraction of the liquid which weeps, liquid Peclet number, stripping factor, point efficiency and the particular Lewis case. Tabulated results are presented from which the apparent Murphree tray efficiency can be determined as a function of each of these five variables. The range of variables covered is of interest to tray designers. For large diameter trays, the liquid Peclet number is high and conditions approximate to plug flow. Analytical solutions are given for this case suitable for inclusion in tray design computer programs. The significance of the work is that, coupled with prediction methods for weeping fraction and point efficiency under weeping conditions, it should now be possible to predict tray efficiency when weeping occurs.

INTRODUCTION

Kageyama (1966, 1969) presented the results of a model from which the effect of weeping on Murphree tray efficiency was calculated. The conditions approximated to those for Lewis' Case 1 (1936). That is, the vapor was completely mixed and the weeping liquid was unmixed between trays. This model is only relevant to small pilot-plant-scale columns in which complete transverse vapor mixing between trays can reasonably be assumed. The only other related study is that of O'Brien (1966), but in his study liquid mixing on the tray was not considered. The present work addresses the problem of the effect of weeping on tray efficiency for all three cases proposed by Lewis. Both analytical and numerical solutions to the problem have been obtained.

The three cases considered are:

- *Case 1:* liquid flows in alternate directions on successive trays, liquid on the tray is partially mixed, vapor is completely mixed between trays, weeping liquid is unmixed between trays.
- *Case 2:* liquid flows in the same direction on successive trays (parallel flow), liquid on the tray is partially mixed, vapor and weeping liquid are unmixed between trays.
- *Case 3:* liquid flows in alternate directions on successive trays, liquid on the tray is partially mixed, vapor and weeping liquid are unmixed between trays.

Case 1 corresponds to small-diameter columns as indicated above. Case 2 corresponds to high-efficiency parallel flow trays as used, for example, by Union Carbide (Smith and Delnicki, 1975) and recently by other tray designers (Haselden et al., 1982; Jenkins, 1981). Case 3, which gives the lowest tray efficiency, corresponds to the majority of trays installed.

For Case 1, simultaneous assumptions are made of no mixing of the weeping liquid but complete vapor mixing between trays. This seems reasonable because of the large size of the weeping drops. In a previous study dealing with entrainment (Lockett et al., 1983, we have considered Case 1 for entrainment to involve complete mixing of both entrained drops and vapor between trays. In view of the generally smaller drop size obtained during entrainment than during weeping, this seems a justifiable assumption. An even more rigorous analysis than that presented below would allow for partial mixing both of weeping liquid and vapor. The benefits to be gained from such a study are questionable, however, because the extent of mixing is not known and the results would differ only marginally from those given in this paper.

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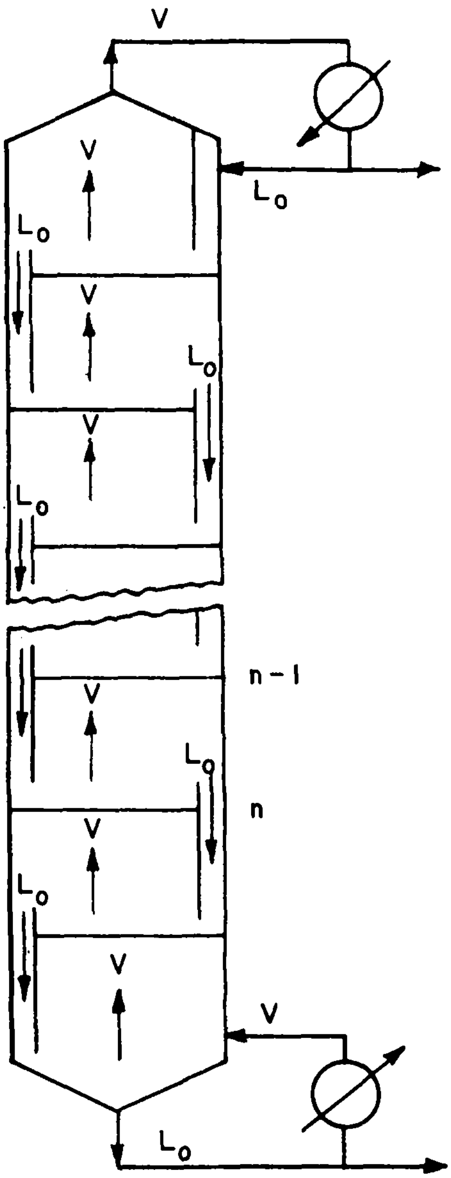

(EQUIMOLAR OVERFLOW)

Figure 1a. Flow rates without weeping (equimolar overflow).

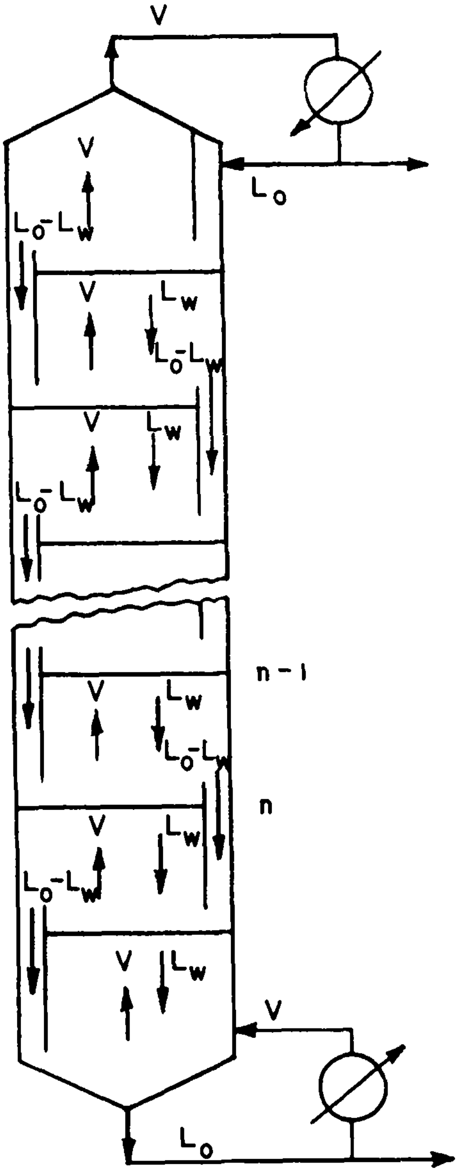

Figure 1b. Flow rates with weeping.

NUMERICAL SOLUTIONS

Main Assumptions

1. Constant liquid, vapor and weeping flow rates from tray to tray.
2. Weeping uniformly distributed over the tray.
3. Constant point efficiency.
4. No mass transfer between vapor and weeping liquid—or if any occurs, it is included in the point efficiency.
5. Rectangular tray.
6. Linear equilibrium relationship $y^{*}_{n} = mx_n + b$

Kageyama (1966) has given the basic differential equation for the liquid concentration on a general tray n as a function of distance from inlet weir, z'.

$$\frac{1}{Pe}\left[\frac{d^2x_n}{dz^2}\right] - \frac{dx_n}{dz} + \beta(x_{n-1} - x_n) - \lambda E(x_n - x^{*}_{e_{n+1}}) = 0 \quad (1)$$

where

$$Pe = \frac{LZ_o}{Wh_f\rho'_L\rho_F De}, \quad \beta = \frac{L_w}{L}, \quad z = \frac{z'}{Z_o}, \quad \lambda = \frac{mV}{L}$$

The four terms in Eq. 1 represent transport of material from or to a point on the tray by the following mechanisms respectively: liquid mixing, flow of liquid, and weeping and mass transfer with the vapor.

As shown in Figure 1, the liquid flow rate on each tray, L, in the presence of weeping, is less than the externally supplied reflux flow rate L_o. It follows that $L = L_o - L_w$. It is convenient to redefine the stripping factor and the weeping fraction in terms of the external flow rates, i.e.,

$$\lambda_o = \frac{mV}{L_o} \quad \text{and} \quad \beta_o = \frac{L_w}{L_o}$$

So that Eq. 1 becomes:

$$\frac{1}{Pe}\left[\frac{d^2x_n}{dz^2}\right] - \frac{dx_n}{dz} + \left(\frac{\beta_o}{1-\beta_o}\right)(x_{n-1} - x_n) - \frac{\lambda_o}{1-\beta_o}E(x_n - x^{*}_{e_{n+1}}) = 0 \quad (2)$$

The usual boundary conditions are used for Eq. 2, i.e.,

a) Danckwerts boundary condition at the inlet weir $z = o$ (Danckwerts, 1953),

$$x_{+n} = x_{1n-1} + \frac{1}{Pe}\left[\frac{dx_n}{dz}\right] \quad (3)$$

b) At the exit weir, $z = 1$,

$$\frac{dx_n}{dz} = 0 \tag{4}$$

The calculation procedure involved the following steps:

(i) Values of λ_o, E, Pe and β_o were set.

(ii) Arbitrary values of x^*_{eR} and $(x_N)_{out}$ were set such that $x^*_{eR} < (x_N)_{out}$, where x^*_{eR} is the liquid composition in equilibrium with the vapor entering the bottom tray from the reboiler (assumed completely mixed) and $(x_N)_{out}$ is the liquid composition leaving the bottom tray. The calculated efficiencies are independent of these assumed values.

(iii) Equation 2 was integrated numerically to obtain the liquid concentration profile on the bottom tray. For the first iteration, β_o was set equal to zero. From the definition of point efficiency and the equilibrium relationship

$$x^*_{en} = x^*_{en+1} + E(x_n - x^*_{en+1})$$

and this equation was used to obtain the x^*_{en} profile leaving the bottom tray.

(iv) The liquid concentration leaving the second tray was determined from Eq. 3.

(v) The vapor concentration profile entering the next tray was determined from the vapor concentration profile leaving the froth of the tray below according to the Lewis case assumed.

(vi) The calculations were repeated tray by tray up the column for a total of ten trays.

(vii) The liquid concentration profiles so obtained were used in the next overall column iteration to estimate the concentration profile of the weeping liquid leaving each tray. The whole procedure was repeated with a finite value of β_o until convergence was achieved on the liquid concentration profiles on each tray.

Calculation of Tray Efficiency

In the presence of weeping, two alternative definitions of tray efficiency can be used. These are the apparent efficiency, E^a_{MV}, analogous to that proposed by Colburn (1936) in the presence of entrainment, and the reduced efficiency, E^r_{MV}, proposed by Standart and Kastanek (1966). These have been proved to be numerically identical in the case of entrainment (Lockett et al., 1983; Rahman and Lockett, 1981), and an exactly analogous proof can be made to show that they are also numerically identical in the presence of weeping. This was also confirmed using the calculated concentrations from the computer program. The definitions of the two efficiencies are:

$$E^a_{MV} = \frac{\overline{Y}_n - \overline{Y}_{n+1}}{y^*_n - \overline{Y}_{n+1}} \tag{5}$$

where

$$y^*_n = m(\bar{x}_n)_{out} + b$$

and

$$\overline{Y}_n = \bar{y}_n + \frac{L_w}{V}\left[(\bar{x}_{n-1})_{out} - \bar{x}_{n-1}\right]$$

and also

$$E^r_{MV} = \frac{\bar{y}_n - \bar{y}_{n+1}}{y^{r*}_n - \bar{y}_{n+1}} \tag{6}$$

where

$$y^{r*}_n = m(\bar{x}^r_n)_{out} + b$$

and

$$(\bar{x}^r_n)_{out} = (\bar{x}_n)_{out} - \frac{L_w}{L_o}\left((\bar{x}_n)_{out} - \bar{x}_n\right)$$

The reduced concentration $(\bar{x}^r_n)_{out}$ is, in fact, the average concentration of the liquid leaving tray n. Either E^a_{MV} or E^r_{MV} can be used in place of the usual Murphree vapor-phase tray efficiency to give the effective efficiency in the presence of weeping. From that point of view, the tray designer in practice need not concern himself with how they are defined, but simply use them as he would a normal Murphree tray efficiency. In his work Kageyama (1966) used E^r_{MV}. However, since E^r_{MV} and E^a_{MV} are numerically identical, we prefer to refer to the efficiency as the apparent efficiency. This then makes the terminology consistent with what we have used for entrainment and as originally used by Colburn (1936).

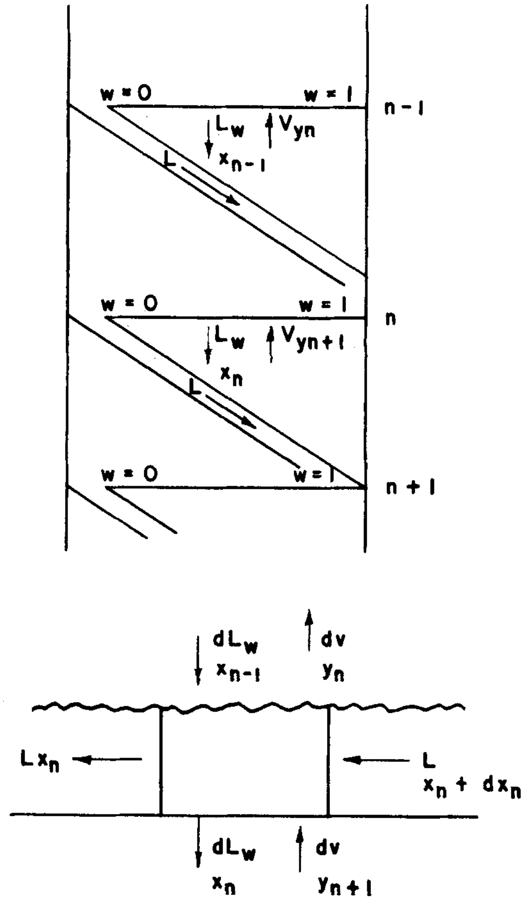

Figure 2. Nomenclature for Lewis Case 2 with weeping.

The apparent tray efficiency was determined from the calculated concentrations around each tray. As the bottom tray receives completely mixed vapor, this has a different efficiency than the others for Lewis Cases 2 and 3. However, by the fourth tray from the bottom of the column, the tray efficiency becomes constant and the reported efficiencies are those for the sixth tray.

ANALYTICAL SOLUTIONS FOR PLUG FLOW OF LIQUID

Lewis Case 2

From a component material balance around a differential slice of the dispersion on tray n, Figure 2, we have

$$[(y_n - y_{n+1}) - f(x_{n-1} - x_n)]Vdw = Ldx_n \tag{7}$$

where $f = L_w/V$ and $dw = dV/V$

Using the equilibrium relationship, $y^* = mx + b$, and with $\lambda = mV/L$, Eq. 7 becomes,

$$\lambda\left[(y_n - y_{n+1}) - \frac{f}{m}(y^*_{n-1} - y^*_n)\right] = \frac{dy^*_n}{dw} \tag{8}$$

The point efficiency E is

$$E = (y_n - y_{n+1})/(y^*_n - y_{n+1}) \tag{9}$$

Substituting and simplifying,

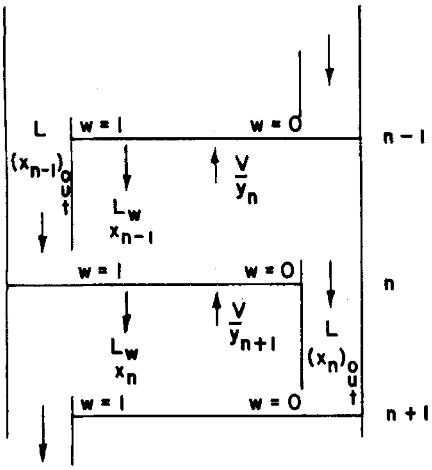

Figure 3. Nomenclature for Lewis Case 3 with weeping.

$$\lambda\left[\left(\frac{Em - Ef + f}{Em}\right)(y_n - y_{n+1}) - \frac{f}{Em}(y_{n-1} - y_n)\right] = \frac{dy_n^*}{dw} \tag{10}$$

Differentiating Eq. 9 with respect to w,

$$\frac{dy_n^*}{dw} = \frac{1}{E}\frac{dy_n}{dw} - \left(\frac{1 - E}{E}\right)\frac{dy_{n+1}}{dw} \tag{11}$$

Substituting Eq. 11 into Eq. 10

$$\lambda\left[\left(\frac{Em - Ef + f}{m}\right)(y_n - y_{n+1}) - \frac{f}{m}(y_{n-1} - y_n)\right] = \frac{dy_n}{dw} - (1 - E)\frac{dy_{n+1}}{dw} \tag{12}$$

This equation corresponds to Lewis' Eq. 3 when $f = 0$ (Lewis, 1936).

Following Lewis we define the usual similarity relations,

$$Z_n = y_n - (y_n)_0 \tag{13}$$

$$Z_{n+1} = y_{n+1} - (y_{n+1})_0$$

$$k_n = (y_n)_0 - (y_{n+1})_0 \tag{14}$$

$$k_{n+1} = (y_{n+1})_0 - (y_{n+2})_0$$

Because of the similarity of concentration profiles on adjacent trays we also have

$$\alpha = Z_n/Z_{n+1} = k_n/k_{n+1} \tag{15}$$

On substituting Eqs. 13–15 into Eq. 12 and simplifying we find,

$$\frac{dZ_n}{(\alpha - 1)Z_n + k_{n-1}} = \frac{\lambda(Em - Ef + f - \alpha f)}{m(\alpha - 1 + E)}dw \tag{16}$$

Integrating Eq. 16 from $w = 0$, $Z_n = 0$ to $w = w$, $Z_n = Z_n$ gives after rearranging

$$ln\left[\frac{(\alpha - 1)Z_n + k_{n-1}}{k_{n-1}}\right] = \frac{(\alpha - 1)\lambda(Em - Ef + f - \alpha f)w}{m(\alpha - 1 + E)} \tag{17}$$

Lewis (1936) showed that for parallel liquid flow $k_n = (Z_{n+1})_1$. The same relation has been shown to hold in the presence of entrainment (Lockett et al., 1983). An exactly similar proof can be used to show that it also holds in the presence of weeping.

Consequently, we have $k_{n-1} = (Z_n)_1$ so that Eq. 17 becomes

$$\lambda = \left[\frac{m(\alpha - 1 + E)}{(\alpha - 1)(Em - f(\alpha - 1 + E))}\right] ln\,\alpha \tag{18}$$

It is generally more convenient to work in terms of the external liquid flow rate, so that with $\beta_o = L_w/L_o$ we have $\lambda = \lambda_o/(1 - \beta_o)$ and $f = m\beta_o/\lambda_o$

On substituting into Eq. 18,

$$\lambda_o = \frac{1}{E}\left[\frac{(\alpha - 1 + E)(1 - \beta_o)ln\,\alpha}{(\alpha - 1)} + \beta_o(\alpha - 1 + E)\right] \tag{19}$$

Equation 19 is the final working equation for Lewis Case 2. It may be used to determine α for particular values of λ_o, E and β_o. Before dealing with how this is then used to determine efficiency, we will derive equivalent equations for Lewis Case 3.

Lewis Case 3

For this case, we define the following similarity relations following Rahman and Lockett (1981). The nomenclature is shown in Figure 3.

$$Z_n = y_n - (y_n)_0 \tag{20}$$

$$Z_{n+1} = y_{n+1} - (y_{n+1})_1$$

$$k_n = (y_n)_0 - (y_{n+1})_1 \tag{21}$$

$$k_{n+1} = (y_{n+1})_1 - (y_{n+2})_0$$

$$\alpha = \frac{(Z_n)_w}{(Z_{n+1})_{1-w}} = \frac{k_n}{k_{n+1}} \tag{22}$$

The basic differential–difference equation, Eq. 12, also holds for this case and substituting Eqs. 20–22 gives:

$$\lambda\left[\frac{\alpha(Em - Ef + 2f)}{m}(Z_n)_w - \left(\frac{Em - Ef + f + \alpha^2 f}{m}\right)(Z_n)_{1-w} + \alpha\left(\frac{Em - Ef + f}{m}\right)k_n - \left(\frac{\alpha f}{m}\right)k_{n-1}\right] = \alpha\left(\frac{dZ_n}{dw}\right)_w + \left[(1 - E)\frac{dZ_n(w)}{dw}\right]_{1-w} \tag{23}$$

Equation 23 can be written as

$$\lambda[A(Z_n)_w - B(Z_n)_{1-w} + C_n] = \alpha\left(\frac{dZ_n}{dw}\right)_w + (1 - E)\left(\frac{dZ_n(w)}{dw}\right)_{1-w} \tag{24}$$

Where

$$A = \frac{\alpha(Em - Ef + 2f)}{m}$$

$$B = \frac{Em - Ef + f + \alpha^2 f}{m} \tag{25}$$

$$C_n = \alpha\left(\frac{Em - Ef + f}{m}\right)k_n - \frac{\alpha f}{m}k_{n-1}$$

Following Lewis, Eq. 24 may be solved by shifting the origin from $w = 0$ to $w = 0.5$. Thus setting $s = w - 0.5$ and

$$u(s) = (Z_n)_w - \frac{C_n}{A - B} \tag{26}$$

Equation 24 becomes

$$\lambda[Au(s) - Bu(-s)] = \alpha\left(\frac{du(s)}{ds}\right)_s + (1 - E)\left(\frac{du(s)}{ds}\right)_{-s} \tag{27}$$

We now assume a solution to Eq. 27 of the form,

$$u = a\,(\sin(\lambda\theta s) - \gamma\cos(\lambda\theta s)) \tag{28}$$

where a is an arbitrary coefficient.

Equation 28 is substituted into Eq. 27 and setting the coefficients of the trigonometric functions to zero, so that the equation is satisfied for all s, enables θ and γ to be determined. The remainder of the mathematical development exactly parallels that given previously for entrainment (Rahman and Lockett, 1981). The final working equation, after converting to the external liquid flow rate as a basis, is

TABLE 1. APPARENT TRAY EFFICIENCY E^a_{MV} FOR LEWIS CASE 1

Pe/λ_o	$\beta_o = 0$: 0.5	1.0	2.0	3.0
		$E = 0.4$		
0	0.40	0.40	0.40	0.40
2	0.42	0.44	0.48	0.52
10	0.43	0.47	0.56	0.66
20	0.44	0.48	0.58	0.71
1,000	0.44	0.49	0.61	0.77
		$E = 0.6$		
0	0.60	0.60	0.60	0.60
2	0.64	0.68	0.77	0.87
10	0.68	0.77	0.99	1.28
50	0.70	0.81	1.12	1.57
1,000	0.70	0.82	1.16	1.68
		$E = 0.8$		
0	0.80	0.80	0.80	0.80
2	0.87	0.95	1.12	1.31
10	0.94	1.12	1.57	2.20
50	0.97	1.20	1.86	2.98
1,000	0.98	1.22	1.97	3.32
		$E = 1.0$		
0	1.00	1.00	1.00	1.00
2	1.11	1.24	1.51	1.84
10	1.23	1.52	2.32	3.55
50	1.28	1.67	2.94	5.38
1,000	1.30	1.72	3.18	6.30

Pe/λ_o	$\beta_o = 0.1$: 0.5	1.0	2.0	3.0
		$E = 0.4$		
0	0.40	0.40	0.40	0.40
2	0.42	0.43	0.47	0.50
10	0.43	0.46	0.53	0.58
20	0.44	0.47	0.54	0.61
1,000	0.44	0.48	0.56	0.64
		$E = 0.6$		
0	0.60	0.60	0.60	0.60
2	0.63	0.67	0.74	0.81
10	0.67	0.74	0.88	1.00
50	0.68	0.77	0.95	1.10
1,000	0.68	0.78	0.97	1.12
		$E = 0.8$		
0	0.80	0.80	0.80	0.80
2	0.86	0.93	1.05	1.16
10	0.92	1.06	1.30	1.48
50	0.94	1.12	1.42	1.61
1,000	0.95	1.14	1.46	1.64
		$E = 1.0$		
0	1.00	1.00	1.00	1.00
2	1.09	1.20	1.39	1.55
10	1.18	1.41	1.77	1.98
50	1.22	1.51	1.95	2.11
1,000	1.23	1.55	2.00	2.14

Pe/λ_o	$\beta_o = 0.3$: 0.5	1.0	2.0	3.0
		$E = 0.4$		
0	0.40	0.40	0.40	0.40
2	0.41	0.42	0.44	0.46
10	0.42	0.44	0.47	0.49
20	0.42	0.44	0.48	0.50
1,000	0.43	0.45	0.48	0.50
		$E = 0.6$		
0	0.60	0.60	0.60	0.60
2	0.63	0.65	0.69	0.71
10	0.65	0.69	0.74	0.77
50	0.66	0.70	0.76	0.77
1,000	0.66	0.71	0.77	0.78
		$E = 0.8$		
0	0.80	0.80	0.80	0.80
2	0.84	0.88	0.94	0.98
10	0.89	0.95	1.03	1.05
50	0.90	0.98	1.05	1.04
1,000	0.91	0.99	1.05	1.04
		$E = 1.0$		
0	1.00	1.00	1.00	1.00
2	1.07	1.13	1.21	1.26
10	1.13	1.23	1.32	1.31
50	1.16	1.28	1.34	1.27
1,000	1.17	1.29	1.34	1.25

Pe/λ_o	$\beta_o = 0.5$: 0.5	1.0	1.5	2.0
		$E = 0.4$		
0	0.40	0.40	0.40	0.40
2	0.41	0.41	0.42	0.42
10	0.41	0.42	0.43	0.43
20	0.41	0.42	0.43	0.43
1,000	0.41	0.42	0.43	0.43
		$E = 0.6$		
0	0.60	0.60	0.60	0.60
2	0.62	0.63	0.64	0.64
10	0.63	0.64	0.65	0.66
50	0.63	0.65	0.66	0.66
1,000	0.63	0.65	0.66	0.65
		$E = 0.8$		
0	0.80	0.80	0.80	0.80
2	0.83	0.85	0.86	0.87
10	0.85	0.87	0.88	0.88
50	0.86	0.88	0.88	0.87
1,000	0.86	0.88	0.88	0.79
		$E = 1.0$		
0	1.00	1.00	1.00	1.00
2	1.04	1.07	1.09	1.10
10	1.08	1.11	1.11	1.11
50	1.09	1.12	1.11	1.08
1,000	1.09	1.12	1.10	1.03

$$\lambda_o = 2(1-\beta_o)\left[\frac{(\alpha^2-(1-E)^2)}{\left(E+\frac{\beta_o}{\lambda_o}(\alpha^2-E+1)\right)^2-\left(\alpha\left\{E+\frac{\beta_o}{\lambda_o}(2-E)\right\}\right)^2}\right]^{0.5}$$

$$\tan^{-1}\frac{\sqrt{\left(\left(E+\frac{\beta_o}{\lambda_o}(\alpha^2-E+1)\right)^2-\left(\alpha\left(E+\frac{\beta_o}{\lambda_o}(2-E)\right)\right)^2\right)(\alpha^2-(1-E)^2)}}{(2\alpha(2-E))\left(E-\frac{\beta_o}{\lambda_o}(\alpha-1+E)\right)-(\alpha-1+E)\left(\left(E+\frac{\beta_o}{\lambda_o}(\alpha^2-E+1)\right)-\left(\alpha\left(E+\frac{\beta_o}{\lambda_o}(2-E)\right)\right)\right)} \tag{29}$$

Equation 29 can be used to determine α for specified values of E, β_o and λ_o. It is used when α (and λ_o) ≤ 1.

To avoid imaginary numbers when $\alpha > 1$, another solution to Eq. 27 may be obtained by assuming a solution of the form

$$u = c\,(\sinh(\lambda\theta s) - \gamma\cosh(\lambda\theta s)). \tag{30}$$

An exactly parallel development leads to Eq. 31

TABLE 2. APPARENT TRAY EFFICIENCY E^a_{MV} FOR LEWIS CASE 2

	$\beta_o = 0$			
Pe/λ_o	0.5	1.0	2.0	3.0
		$E = 0.4$		
0	0.40	0.40	0.40	0.40
2	0.42	0.44	0.48	0.52
10	0.44	0.48	0.57	0.68
20	0.44	0.49	0.60	0.74
1,000	0.45	0.50	0.63	0.81
		$E = 0.6$		
0	0.60	0.60	0.60	0.60
2	0.64	0.69	0.79	0.89
10	0.69	0.80	1.05	1.37
50	0.71	0.84	1.20	1.72
1,000	0.71	0.86	1.25	1.85
		$E = 0.8$		
0	0.80	0.80	0.80	0.80
2	0.88	0.97	1.15	1.36
10	0.98	1.19	1.73	2.46
50	1.02	1.30	2.11	3.45
1,000	1.03	1.33	2.25	3.89
		$E = 1.0$		
0	1.00	1.00	1.00	1.00
2	1.13	1.28	1.59	1.95
10	1.32	1.70	2.68	4.14
50	1.41	1.93	3.55	6.58
1,000	1.43	2.00	3.90	7.82

	$\beta_o = 0.1$			
Pe/λ_o	0.5	1.0	2.0	3.0
		$E = 0.4$		
0	0.40	0.40	0.40	0.40
2	0.42	0.44	0.47	0.51
10	0.43	0.47	0.55	0.63
20	0.44	0.49	0.58	0.66
1,000	0.44	0.49	0.59	0.71
		$E = 0.6$		
0	0.60	0.60	0.60	0.60
2	0.64	0.68	0.76	0.85
10	0.68	0.77	0.96	1.16
50	0.70	0.81	1.06	1.34
1,000	0.70	0.82	1.10	1.40
		$E = 0.8$		
0	0.80	0.80	0.80	0.80
2	0.87	0.95	1.10	1.25
10	0.96	1.14	1.50	1.88
50	1.00	1.23	1.73	2.26
1,000	1.00	1.25	1.80	2.38
		$E = 1.0$		
0	1.00	1.00	1.00	1.00
2	1.12	1.24	1.49	1.73
10	1.29	1.58	2.19	2.79
50	1.37	1.76	2.61	3.44
1,000	1.39	1.82	2.75	3.65

	$\beta_o = 0.3$			
Pe/λ_o	0.5	1.0	2.0	3.0
		$E = 0.4$		
0	0.40	0.40	0.40	0.40
2	0.41	0.43	0.45	0.47
10	0.43	0.45	0.50	0.55
20	0.43	0.46	0.52	0.56
1,000	0.43	0.47	0.53	0.59
		$E = 0.6$		
0	0.60	0.60	0.60	0.60
2	0.63	0.66	0.72	0.77
10	0.66	0.73	0.83	0.93
50	0.68	0.75	0.89	1.00
1,000	0.68	0.76	0.90	1.02
		$E = 0.8$		
0	0.80	0.80	0.80	0.80
2	0.86	0.91	1.01	1.09
10	0.93	1.04	1.23	1.37
50	0.95	1.10	1.32	1.50
1,000	0.96	1.11	1.35	1.53
		$E = 1.0$		
0	1.00	1.00	1.00	1.00
2	1.09	1.17	1.33	1.45
10	1.23	1.40	1.68	1.87
50	1.29	1.51	1.83	2.06
1,000	1.30	1.54	1.88	2.11

	$\beta_o = 0.5$			
Pe/λ_o	0.5	1.0	1.5	2.0
		$E = 0.4$		
0	0.40	0.40	0.40	0.40
2	0.41	0.42	0.43	0.43
10	0.42	0.44	0.45	0.47
20	0.42	0.44	0.46	0.47
1,000	0.42	0.44	0.46	0.48
		$E = 0.6$		
0	0.60	0.60	0.60	0.60
2	0.62	0.64	0.66	0.68
10	0.64	0.68	0.72	0.75
50	0.65	0.70	0.74	0.77
1,000	0.66	0.71	0.75	0.78
		$E = 0.8$		
0	0.80	0.80	0.80	0.80
2	0.84	0.88	0.91	0.93
10	0.89	0.96	1.01	1.05
50	0.91	0.99	1.05	1.10
1,000	0.91	1.00	1.06	1.11
		$E = 1.0$		
0	1.00	1.00	1.00	1.00
2	1.07	1.12	1.17	1.21
10	1.14	1.26	1.33	1.39
50	1.18	1.32	1.40	1.46
1,000	1.19	1.33	1.42	1.48

$$\lambda_o = 2(1-\beta_o)\left[\frac{(\alpha^2-(1-E)^2)}{\left(\alpha\left(E+\frac{\beta_o}{\lambda_o}(2-E)\right)\right)^2-\left(E+\frac{\beta_o}{\lambda_o}(\alpha^2-E+1)\right)^2}\right]^{0.5} \tanh^{-1}\frac{\sqrt{\left(\left(\alpha\left(E+\frac{\beta_o}{\lambda_o}(2-E)\right)\right)^2-\left(E+\frac{\beta_o}{\lambda_o}(\alpha^2-E+1)\right)^2\right)(\alpha^2-(1-E)^2)}}{(2\alpha(2-E))\left(E-\frac{\beta_o}{\lambda_o}(\alpha-1+E)\right)-(\alpha-1+E)\left(\left(E+\frac{\beta_o}{\lambda_o}(\alpha^2-E+1)\right)-\left(\alpha\left(E+\frac{\beta_o}{\lambda_o}(2-E)\right)\right)\right)} \qquad (31)$$

Equation 31 is used to determine α when α (and λ_o) ≥ 1.

Calculation of Apparent Tray Efficiency

When α has been determined from Eq. 19, Eq. 29 or 31 as appropriate, the apparent Murphree tray efficiency E_{MV} may be determined from the following equation

$$E^a_{MV} = \frac{\alpha - 1}{\lambda_o - 1} \qquad (32)$$

Equation 32 was originally derived by Lewis (1936) in the absence of weeping or entrainment. It has been proved to hold in the

TABLE 3. APPARENT TRAY EFFICIENCY E^a_{MV} FOR LEWIS CASE 3

$\beta_o = 0$

Pe/λ_o	0.5	1.0	2.0	3.0
		$E = 0.4$		
0	0.40	0.40	0.40	0.40
2	0.42	0.44	0.47	0.51
10	0.43	0.47	0.55	0.65
20	0.44	0.48	0.57	0.69
1,000	0.44	0.49	0.60	0.75
		$E = 0.6$		
0	0.60	0.60	0.60	0.60
2	0.64	0.68	0.76	0.85
10	0.68	0.76	0.95	1.20
50	0.69	0.80	1.06	1.44
1,000	0.70	0.81	1.10	1.53
		$E = 0.8$		
0	0.80	0.80	0.80	0.80
2	0.87	0.94	1.09	1.26
10	0.93	1.07	1.44	1.94
50	0.96	1.14	1.65	2.49
1,000	0.97	1.16	1.73	2.72
		$E = 1.0$		
0	1.00	1.00	1.00	1.00
2	1.10	1.20	1.44	1.72
10	1.17	1.39	1.98	2.89
50	1.20	1.47	2.33	3.95
1,000	1.21	1.50	2.46	4.43

$\beta_o = 0.1$

Pe/λ_o	0.5	1.0	2.0	3.0
		$E = 0.4$		
0	0.40	0.40	0.40	0.40
2	0.42	0.43	0.46	0.49
10	0.43	0.46	0.52	0.57
20	0.43	0.47	0.53	0.60
1,000	0.44	0.48	0.55	0.62
		$E = 0.6$		
0	0.60	0.60	0.60	0.60
2	0.63	0.67	0.73	0.79
10	0.67	0.73	0.85	0.96
50	0.68	0.76	0.91	1.03
1,000	0.68	0.77	0.93	1.05
		$E = 0.8$		
0	0.80	0.80	0.80	0.80
2	0.86	0.92	1.02	1.12
10	0.91	1.02	1.21	1.36
50	0.93	1.07	1.29	1.43
1,000	0.94	1.08	1.32	1.44
		$E = 1.0$		
0	1.00	1.00	1.00	1.00
2	1.08	1.17	1.33	1.48
10	1.14	1.30	1.57	1.74
50	1.17	1.35	1.65	1.75
1,000	1.17	1.36	1.64	1.72

$\beta_o = 0.3$

Pe/λ_o	0.5	1.0	2.0	3.0
		$E = 0.4$		
0	0.40	0.40	0.40	0.40
2	0.41	0.42	0.44	0.45
10	0.42	0.44	0.46	0.48
20	0.42	0.44	0.47	0.49
1,000	0.43	0.45	0.48	0.49
		$E = 0.6$		
0	0.60	0.60	0.60	0.60
2	0.62	0.65	0.68	0.71
10	0.64	0.68	0.72	0.74
50	0.65	0.69	0.73	0.74
1,000	0.66	0.70	0.74	0.73
		$E = 0.8$		
0	0.80	0.80	0.80	0.80
2	0.84	0.88	0.93	0.96
10	0.87	0.92	0.97	0.98
50	0.89	0.94	0.97	0.93
1,000	0.89	0.94	0.96	0.91
		$E = 1.0$		
0	1.00	1.00	1.00	1.00
2	1.06	1.11	1.18	1.22
10	1.09	1.15	1.20	1.18
50	1.10	1.16	1.15	1.06
1,000	1.10	1.15	1.14	1.00

$\beta_o = 0.5$

Pe/λ_o	0.5	1.0	1.5	2.0
		$E = 0.4$		
0	0.40	0.40	0.40	0.40
2	0.41	0.41	0.42	0.42
10	0.41	0.42	0.42	0.43
20	0.41	0.42	0.43	0.43
1,000	0.41	0.42	0.43	0.43
		$E = 0.6$		
0	0.60	0.60	0.60	0.60
2	0.62	0.63	0.63	0.64
10	0.62	0.64	0.64	0.64
50	0.63	0.64	0.64	0.63
1,000	0.63	0.64	0.63	0.62
		$E = 0.8$		
0	0.80	0.80	0.80	0.80
2	0.83	0.84	0.85	0.86
10	0.84	0.85	0.85	0.84
50	0.84	0.84	0.82	0.79
1,000	0.84	0.84	0.82	0.76
		$E = 1.0$		
0	1.00	1.00	1.00	1.00
2	1.03	1.05	1.07	1.08
10	1.04	1.05	1.04	1.02
50	1.03	1.02	0.98	0.92
1,000	1.03	1.00	0.95	0.86

presence of entrainment (Rahman and Lockett, 1981; Lockett et al., 1983), and an exactly similar proof shows it to hold also for weeping. It was in fact used previously by Kageyama (1966) for Lewis Case 1 weeping.

TABULATED RESULTS

The numerical and analytical solutions were used to construct Tables 1–3. These show the apparent tray efficiency E^a_{MV} as a function of the Lewis case, point efficiency E, stripping factor λ_o, liquid Peclet number and fractional weeping β_o.

DISCUSSION

The effect of each of the five variables considered on the apparent tray efficiency is shown graphically in Figures 4–7.

Figure 4 shows the effect of weeping fraction for typical values of E, λ_o and Pe found in distillation. It is assumed here that the point efficiency does not change as weeping increases. It is clear that the efficiency advantage of Lewis Case 2 is maintained over the other two cases even in the presence of substantial weeping.

Figure 5 shows the effect of Peclet number, and indirectly of tray size, when weeping occurs. For a small tray, or a short flow path length, giving a low Peclet number, weeping causes only a small reduction in tray efficiency. This can be understood by considering the limiting case of a perfectly mixed tray ($Pe = 0$). It is then immaterial how the liquid on the tray progresses to the tray below. Both weeping liquid and liquid passing down the downcomer have the same concentration; so in this case weeping causes no reduction in efficiency. This is the main difference between the effect of entrainment and weeping on tray efficiency. Entrainment causes a reduction in tray efficiency even for a perfectly mixed tray which can be calculated from Colburn's (1936) equation.

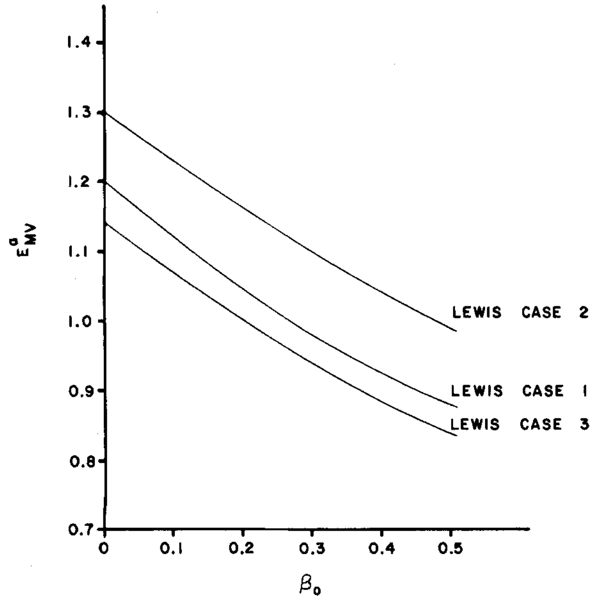

Figure 4. Apparent tray efficiency vs. weeping fraction for E = 0.8, λ_o = 1.0 and Pe = 50.

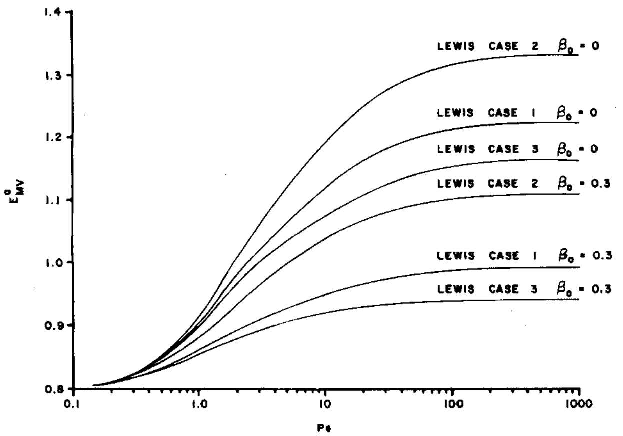

Figure 5. Apparent tray efficiency vs. Peclet Number for λ_o = 1.0, E = 0.8.

The total reduction in tray efficiency caused by weeping will generally be larger than that shown in Figures 4 and 5, because they are plotted for a constant value of point efficiency E. Since weeping reduces the flow rate of liquid flowing over the weir, the froth height is reduced and so also is the point efficiency. The extent of the latter depends on the relative magnitudes of the liquid crest height over the weir and the weir height, and so will vary from case to case. To take this into account the calculated results given in Tables 1–3 must be combined with a model to predice point efficiency under weeping conditions.

Figure 6 shows the effect of large values of the stripping factor such as may be encountered in absorption or stripping. It shows that, as the stripping factor increases, the penalty paid in efficiency from only a small amount of weeping becomes severe. As weeping increases, the tray efficiency is approximately equal to the point efficiency.

Figure 7 shows that, under distillation conditions, weeping has only a small effect when the point efficiency is low. Trays designed for high point efficiency are more severely affected by weeping. In fact all these results show that trays designed to give a high tray efficiency, by exploiting crossflow enhancement of tray efficiency over point efficiency, are particularly vulnerable to a failure to achieve that enhancement as a result of weeping. A tray which is only essentially a point efficiency tray can tolerate much more weeping than one designed to achieve maximum tray efficiency.

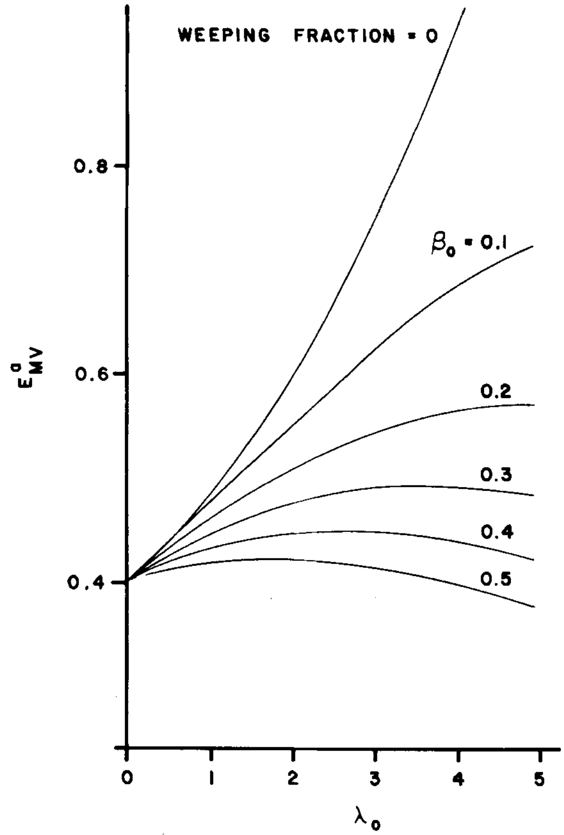

Figure 6. Apparent tray efficiency vs. stripping factor for Lewis Case 3, E = 0.4, Pe = ∞.

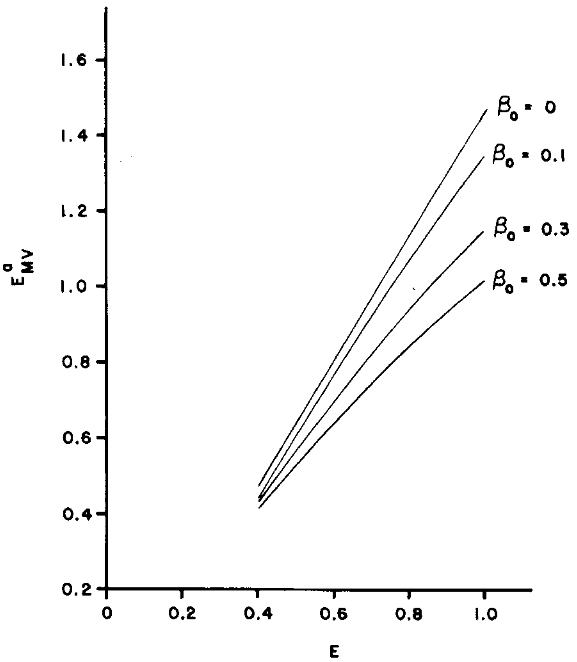

Figure 7. Apparent tray efficiency vs. point efficiency for Lewis Case 3, λ_o = 1.0, Pe = 50.

Even so, a tray designed to exploit enhancement of tray efficiency over point efficiency will still give a higher efficiency than a point efficiency tray even under weeping conditions.

To use the above models it is necessary to predict the weeping fraction. Existing correlations in the literature leave something to be desired and we are presently concluding a comprehensive experimental study on this aspect of the problem. Most designers in practice have access to proprietary correlations, however, such as that of FRI, so this should not prove to be a barrier to incorporation of this present work into tray design methods.

NOTATION

A = variable defined by Eq. 25
a = coefficient in Eqs. 28 and 30
B = variable defined by Eq. 25
b = constant in equilibrium line equation
C_n = variable defined by Eq. 25
D_e = eddy diffusion coefficient of mixing ($m^2 \cdot s^{-1}$)
E = Murphree vapor-phase point efficiency
E^a_{mv} = apparent Murphree vapor-phase tray efficiency
E^r_{mv} = reduced Murphree vapor-phase tray efficiency
f = weeping fraction based on vapor flow L_w/V
h_f = froth height (m)
k_n = defined by Eq. 14 or 21
L = liquid flow rate on tray in presence of weeping = $L_o - L_w$ ($kmol \cdot s^{-1}$)
L_o = liquid flow rate on tray in absence of weeping ($kmol \cdot s^{-1}$)
L_w = weeping liquid flow rate ($kmol \cdot s^{-1}$)
m = slope of equilibrium line
Pe = liquid Peclet number
s = $w - 0.5$
$u(s)$ = function defined by Eq. 26
V = vapor flow rate ($kmol \cdot s^{-1}$)
W = tray width normal to liquid flow (m)
w = position variable on tray defined in Figures 2 and 3
x = liquid concentration (mole fraction)
$\bar{x}_n$ = mean concentration of weeping liquid tray n
$(\bar{x}_n)_{out}$ = mean concentration of liquid leaving tray n via the downcomer
$\bar{x}^r)_{out}$ = reduced liquid concentration leaving tray n
x^*_{en} = liquid concentration in equilibrium with vapor of concentration y_n
x_{+n} = liquid concentration at inlet of tray n
x_{1n} = liquid concentration at inlet weir of tray n
Y = apparent vapor concentration
y = vapor concentration (mole fraction)
y^* = vapor concentration in equilibrium with liquid of concentration x
y^{r*} = vapor concentration in equilibrium with liquid concentration $(\bar{x}^r_n)_{out}$
$(y_n)_0$, $(y_n)_1$ = vapor concentration leaving points 0 and 1 respectively on tray n
Z_n = defined by Eqs. 13 and 20
Z_o = liquid flow path length (m)
z = dimensionless distance z'/Z_o
z' = distance from inlet weir (m)
α = similarity ratio, Eq. 15 or 22
β = L_w/L
β_o = L_w/L_o
γ,θ = constants in Eqs. 28 and 30
λ = stripping factor mV/L
λ_o = stripping factor mV/L_o
ρ_F = holdup fraction of liquid in froth (m^3 liquid/m^3 froth)
ρ'_L = clear liquid density ($kmol \cdot m^{-3}$)

Superscripts

$-$ = mean value

Subscripts

n = on or leaving tray n
N = on or leaving bottom tray
R = leaving reboiler

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Manuscript received February 9, 1983; revision received May 31, and accepted June 10, 1983.